The last 5 years of O.M. Sharkovsky's fruitful work

In 2018 O.M. Sharkovsky completed work on the monograph ***"Ideal Turbulence: fractal and stochastic attractors of trajectories in idealised models of Mathematical Physics"*** co-authored with Romanenko O. Yu.

This monograph was the result of many years of research on the application of one-dimensional dynamical systems theory for the modeling and simulation of distributed (spatio-temporal) chaos in infinite-dimensional dynamical systems.

After completing the work on this monograph, O.M. Sharkovsky again focuses on one-dimensional dynamics, this time in terms of descriptive chaos theory, which becomes a priority for him.

He continued further research into the descriptive theory of deterministic chaos.

Public speaking and lectures

In 2018, he presented papers on **"Descriptive Theory of the deterministic Chaos"** at international scientific conferences in Ukraine:

1) 4th International Conference Nonlinear Analysis and Applications (April 4-6, Kyiv 2018);

2) International Scientific Conference "Contemporary Problems of Mechanics and Mathematics" (January 22-25, Lviv 2018).

In the last 5 years, O.M. Sharkovsky has had 2 trips abroad and public appearances there.

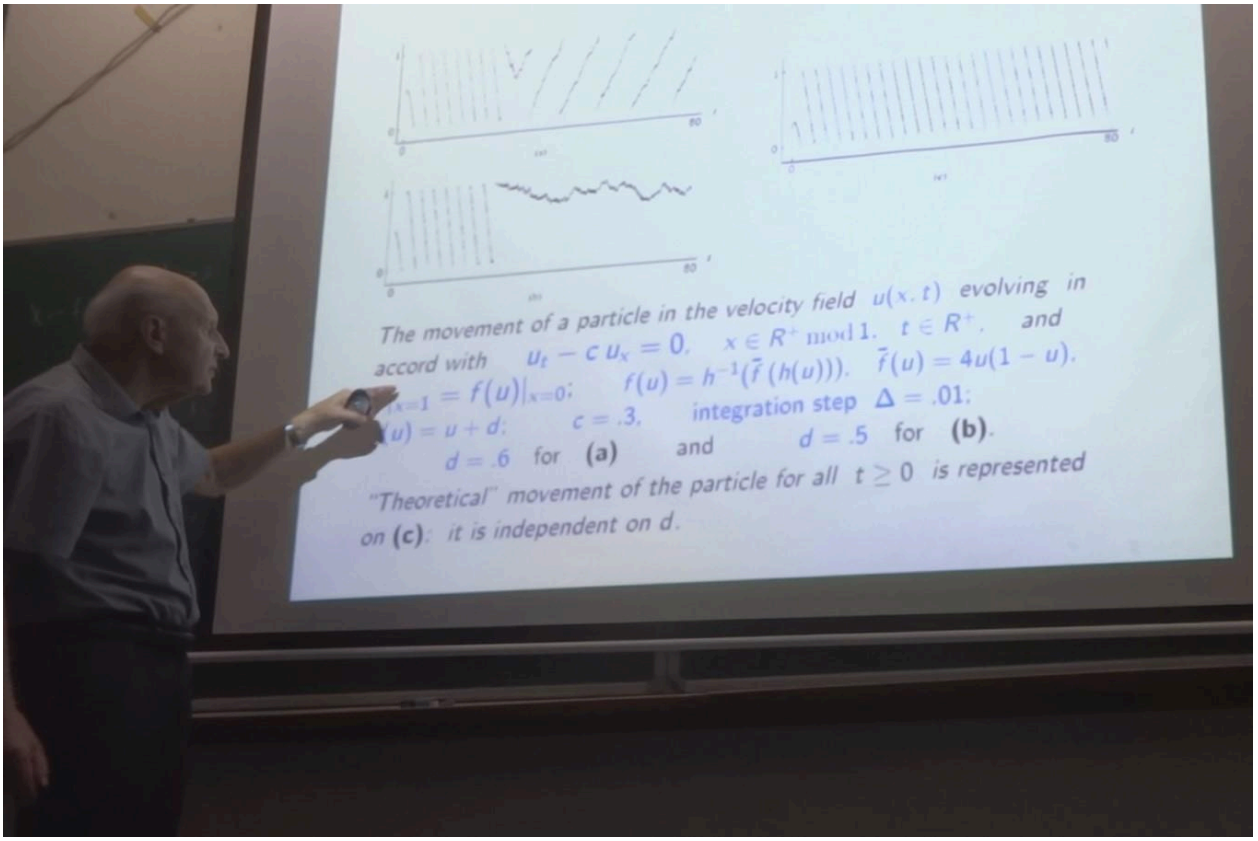
The first trip was to India.

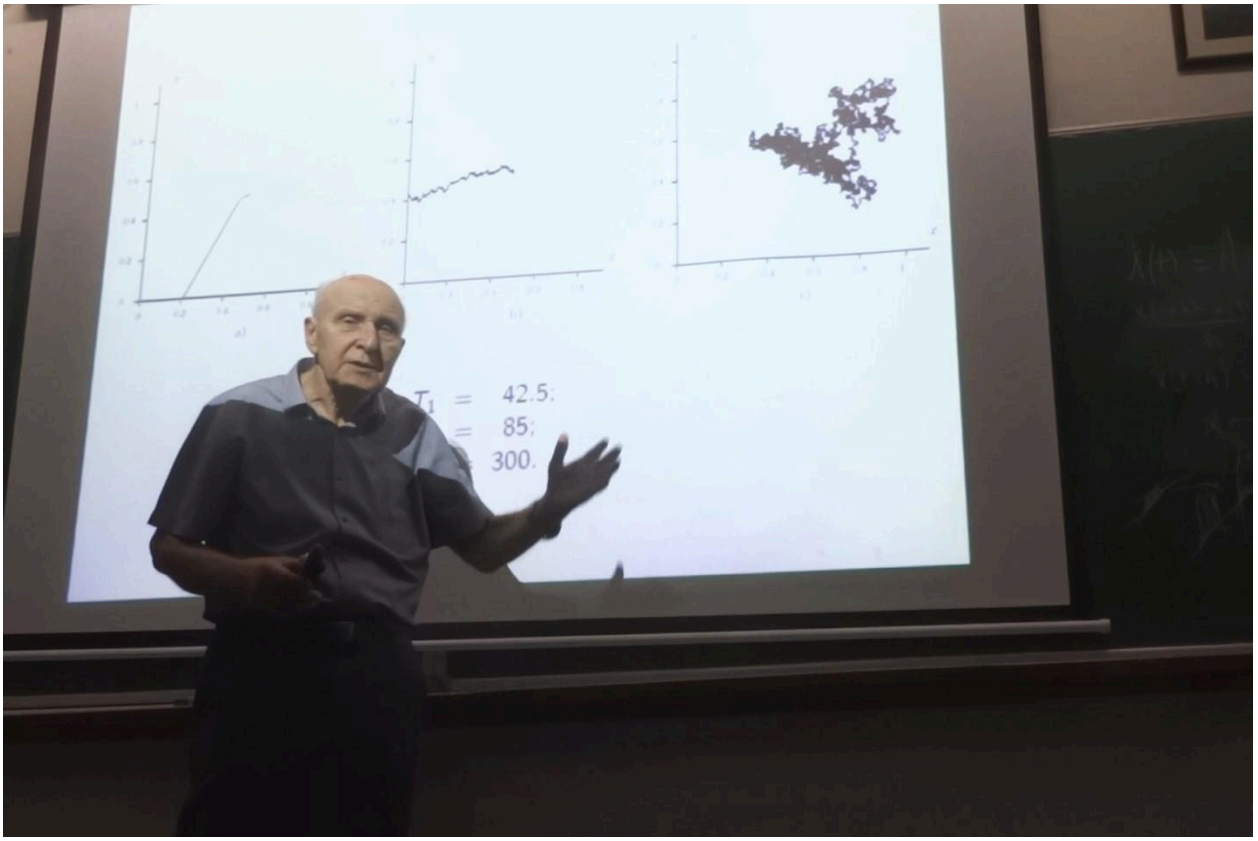
At the end of 2018, he visited India. He gave three lectures at the University of Hyderabad (School of Mathematics and Statistics, University of Hyderabad, India, from 27th November to 2nd December 2018):

1. **"Sharkovsky ordering"**;

2. **"Descriptive theory of chaos: Attractors of trajectories and their Basins";**

3**. "Ideal turbulence".**





Lectures at the University of Hyderabad

Lectures at the University of Hyderabad

Post-lecture discussion

Then, the IWCTA 2018 International Workshop and Conference on Topology and Applications was held in Cochin, Kerala, India, from 5th to 11th December. The list of participants included a number of well-known American experts in the field of dynamical systems and chaotic dynamics, such as J.Auslander, J.Yorke, R.Devaney, and many others.

Oleksandr Sharkovsky gave two talks about **Sharkovsky ordering** at the IWCTA 2018 International Workshop (December 5-8, 2018).

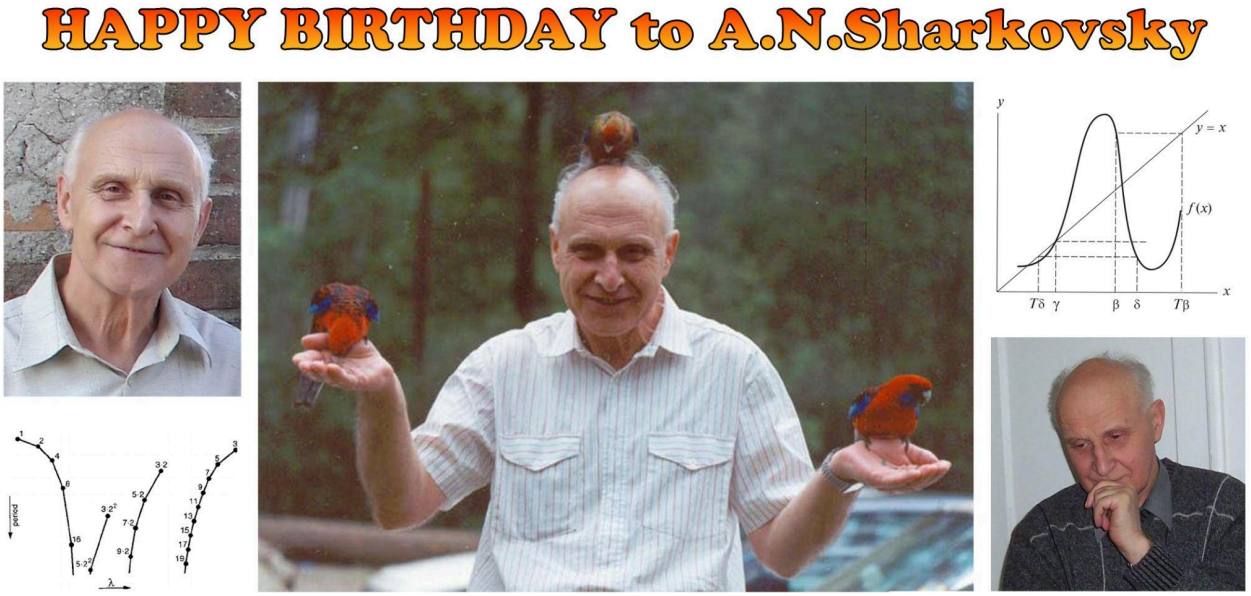
Also, at the IWCTA International Conference (December 9-11, 2018) he gave a presentation on the **"DESCRIPTIVE THEORY OF CHAOS"**, which discussed the application of descriptive set theory to the theory of dynamical systems.

At the IWCTA 2018 International Conference

At the IWCTA 2018 International Conference

His birthday on December 7 fell on one of the days of the conference and the organizers decided to celebrate it together with all the conference participants.

There are a few photos taken during the celebration of O. M. Sharkovsky's birthday.



Prof. James Yorke, University of Maryland, USA

Prof. V. Kannan, University of Hyderabad, India

Prof. V. Kannan (Hyderabad) presents O. M. Sharkovsky with the ritual shawl.

Prof. James Yorke (Maryland) congratulates Oleksandr Sharkovsky.

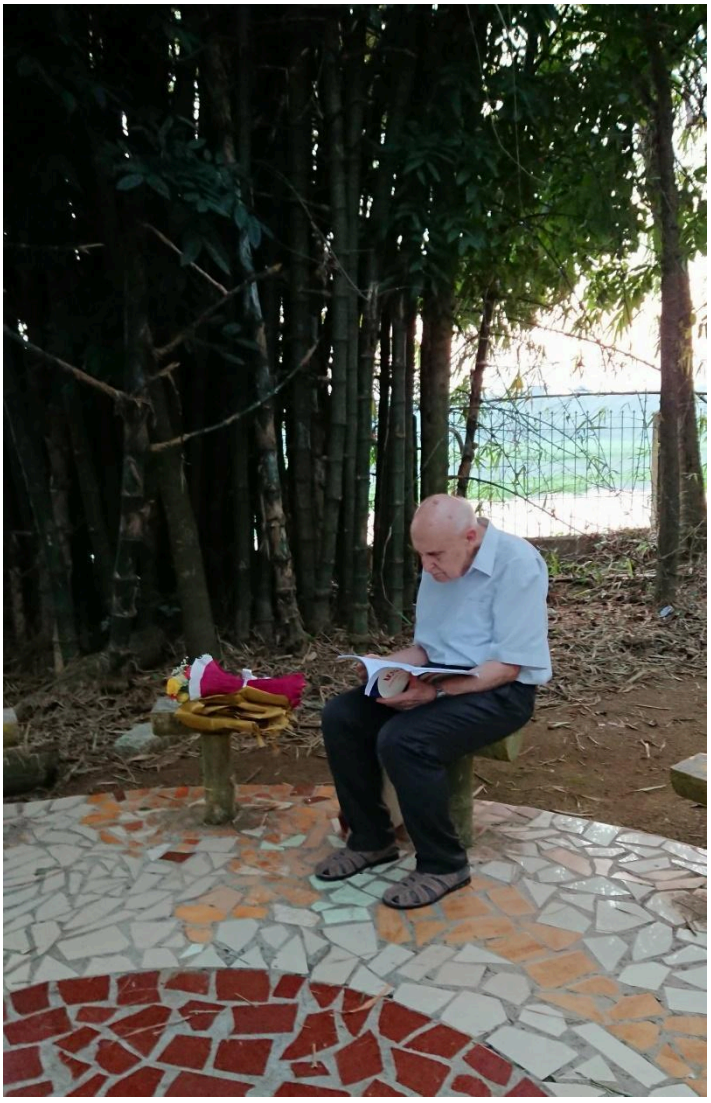
Cutting a birthday cake for all participants after the ceremonial part is over.

Andrei Tetenov, Gorno-Altaisk State University (Russia), Dr. Vinod Kumar P B, RSET (India), Oleksandr Sharkovsky, Olena Sharkovska (Ukraine), Dr.John M. George, RSET (India).

O. M. Sharkovsky with the ritual shawl after the birthday celebration.

A few photos with the conference participants:

On the grounds of the Rajagiri School of Engineering & Technology garden.



Relaxation in the school garden after the ceremonial part of the birthday celebration.

The second trip was to Germany.

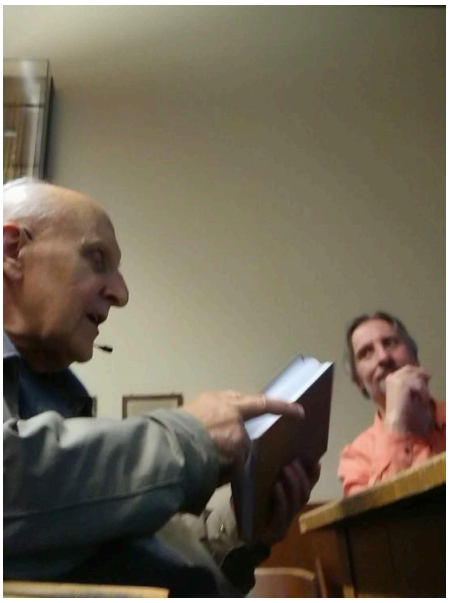
A trip to Germany was from 5 to 12 November 2019 with the invitation of Prof. V. Pidstrygach of the Institute of Mathematics, University of Göttingen. There were given two lectures at the Mathematisches Institut, Georg-August-Universitat Göttingen:

1. **"Descriptive theory of the deterministic chaos: Attractors of trajectories and their Basins."**

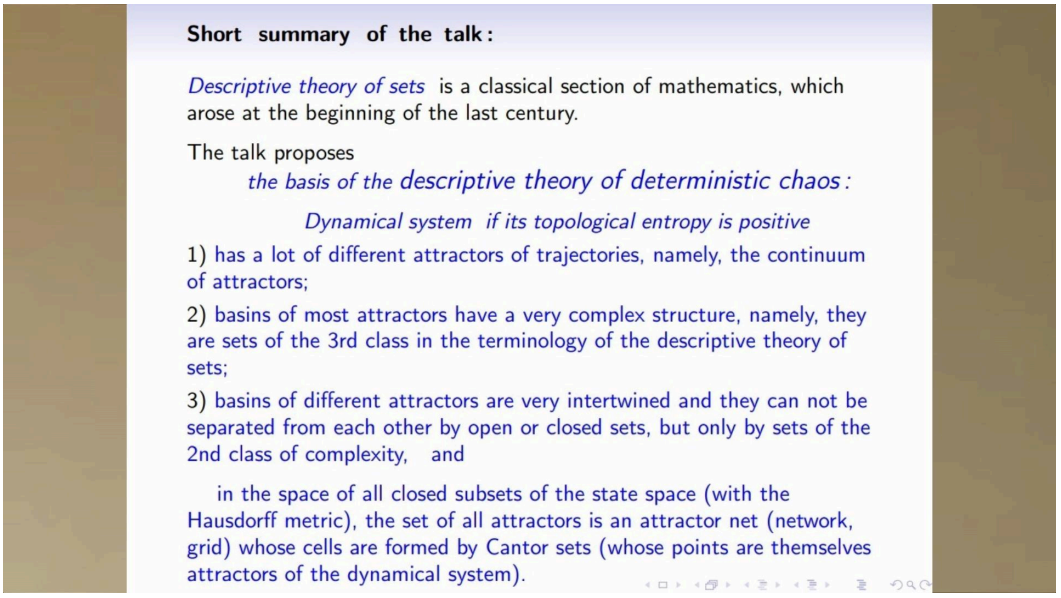
2**. "Sharkovsky ordering and combinatorial dynamics".**

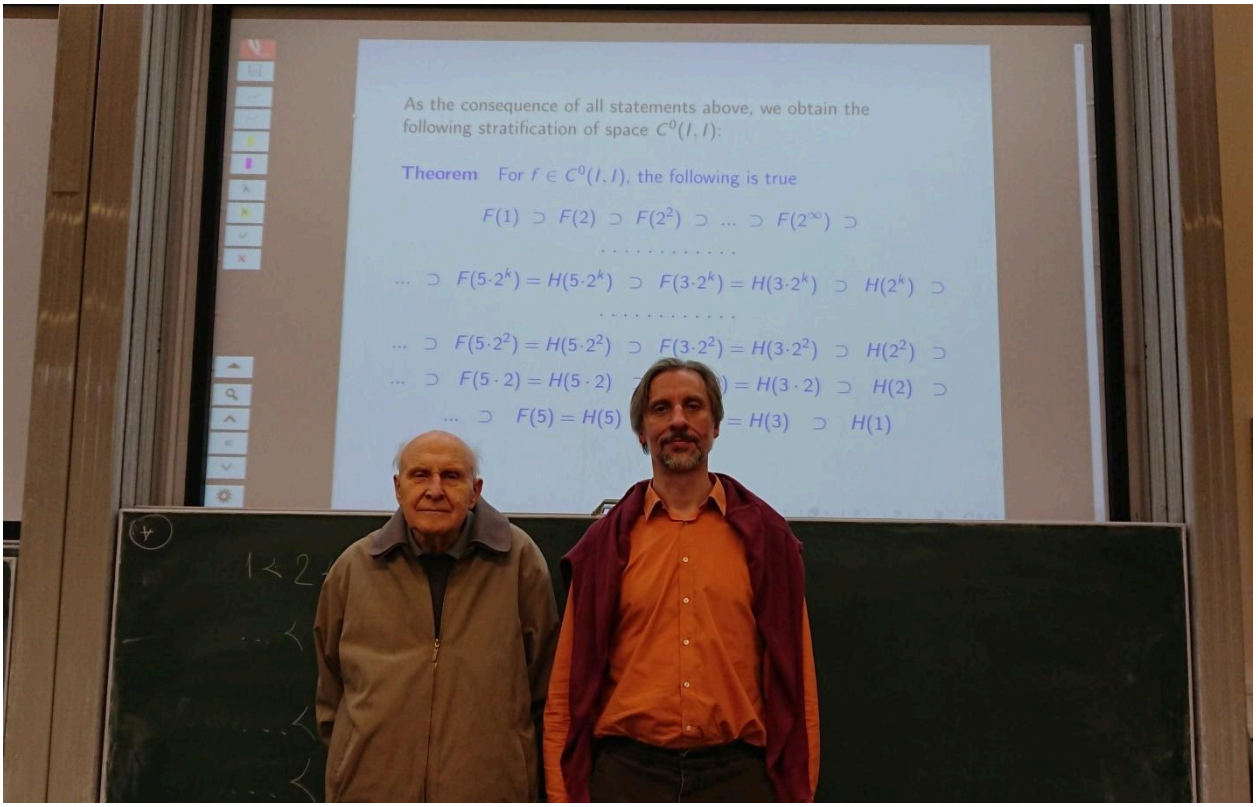
Oleksandr Sharkovsky had to make these two lectures in the auditorium of the University of Göttingen sitting in a chair (it was difficult for him to stand).

On 5th November 2019, during his trip to Göttingen via Frankfurt from Kyiv, he fell at the Frankfurt Airport and damaged his spine. The wheel of a suitcase got caught on the escalator. He fell on his back and ended up with 11 traumatised vertebrae. He only found out about the severity of his injuries after returning to Kyiv.

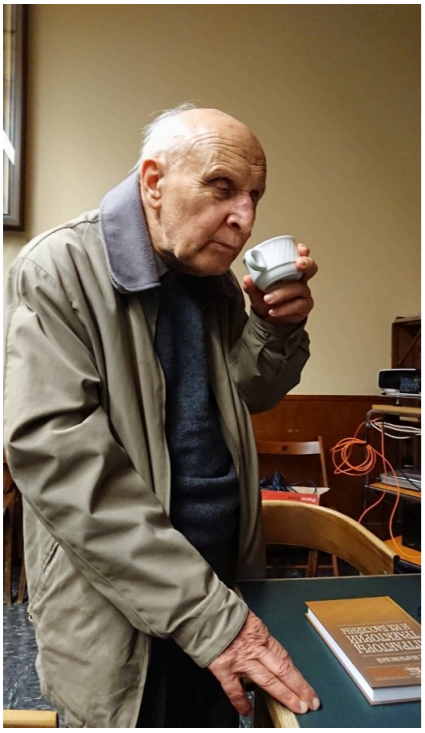


During the lecture…





O. Sharkovsky and V. Pidstrygach



Without interrupting the thought... a cup of coffee after the lecture

The lectures were successfully delivered. During his stay in Göttingen, he spent most of his time in his hotel room, except when he had lectures to attend. At the end of his stay in Göttingen, he wanted to explore the ancient city center. We walked with him for 1,5 hours.

On the Market Square (Marktplatz)
in the old town of Göttingen

Near the monument to the German scientist
Georg Christoph Lichtenberg

It was my father's last trip abroad and his last public appearance in front of an audience.

In 2020 Oleksandr Sharkovsky conducted the 1st academic semester online seminar on the topic **"Scientific Seminar of Mathematical Analysis and Dynamical Systems"** with the students of the 2nd year of the Faculty of Mechanics and Mathematics of Taras Shevchenko National University of Kyiv.

In the spring of 2021 it was planned to perform in front of an audience of students - mathematicians of Taras Shevchenko National University of Kyiv, but due to the COVID-19 quarantine in the country, the lecture was postponed indefinitely.

Publications

The ***"Ideal Turbulence: Fractal and Stochastic Trajectory Attractors in Idealised Models of Mathematical Physics"*** monograph was published by the Institute of Mathematics of NAS of Ukraine in 2020 with co-author E.Yu.Romanenko.

The ***"Computer Turbulence as a Tunneling Effect"*** article was published by the Journal of Mathematical Sciences (vol. 256, pp. 703–712, July 2021) and was written by him with E.Yu.Romanenko, and A.A.Akbergenov.

The ***"Asymptotic Properties of the Semigroup Generated by a Continuous Interval Map"***, article was published by the International Journal of Mathematical, Engineering, Biological and Applied Computing (vol. 1, no. 2, pp. 77-94) in 2022 with co-author E.Yu.Romanenko.

In 2021 he completed a **"Sharkovsky Ordering"** with co-author A. Blokh. This book was published in 2022 by Springer.

From the book: **"Sharkovsky Ordering"**:

«What was the author doing after the paper on the coexistence of periods came out?

Perhaps it would be appropriate to mention and discuss "new surprising and unexpected patterns" which became the focus of his studies.

The paper ***The reducibility of a continuous function of a real variable and the structure of the fix points of the corresponding iteration process***

(Russian), Dokl. Akad. Nauk SSSR 139 (1961) no. 5, 1067-1070 (English)

indicates that already in 1961 the author was interested in not only periodic orbits (cycles), arguably the simplest types of orbits, but also the global behavior of the process of iteration. This allowed him to get to the bottom of the majority of problems of topological dynamics that appear in the one-dimensional case. In particular, it was possible to develop the basis of the descriptive theory of chaos by applying the descriptive theory of sets to studying (and characterizing) of chaotic behavior of trajectories.

It is time to pull from the oblivion the author's results from 1960-1963 (this was more than 50 years ago), and go over the descriptive theory of chaos developed in them! The first here was the paper ***On attracting and attracted sets*** Dokl. Akad. Nauk SSSR 160 (1965), 1036-1038 (Russian), containing upper descriptive estimates of the complexity of basins of attraction of various attractors for dynamical systems in arbitrary compact spaces. This article was presented to the Reports of the

Academy of Sciences of the USSR in 1964 by the academician P. S. Aleksandrov, one of the creators of the descriptive set theory. The author at that time was not yet 28. In the following papers ***A classification of fixed points***, Amer. Math. Soc. Transl. 97 (1970) no. 2, 159-179 (transl. from Ukrain. Mat. Zh. 17 (1965), no. 5, 80-95) and ***Behavior of a mapping in the neighborhood of an attracting set***, Amer. Math. Soc. Transl. (2) 97 (1970), 227-258 (transl. from Ukrain. Mat. Zh. 18 (1966), no. 2, 60-83) published in 1965-66 and soon translated into English by the AMS, it was proven that all these upper estimates are realized for dynamical systems on the real line. From the descriptive set theory point of view this means that dynamics on the line can be just as complex (chaotic) as dynamics on any other compact space! It is worth mentioning here that this direction was not as "lucky" as the other one: if the paper on coexistence of cycles on the line translated into English in 1995 has over 1500 citations, the papers devoted to the other direction studied by the author can hardly boast 100 citations (which is quite understandable as it deals with much more complicated notions that, in addition, are "hidden" behind the titles that were not too revealing)»

For Oleksandr Sharkovsky the direction of research on chaos was very important. After two last foreign trips and performances, he wrote in his letter to Prof. Kloeden:

> *«...My 'walk' to India in November-December 2018 was very successful (J. Yorke was also present at my chaos talk)....»*
>
> *«... I am sending you the slides of one of my two talks that I had in Gottingen last November. This talk is about chaos. Each time at the same time, I recall that the first reaction to J. Yorke's article "Period 3 implies chaos" was your note in "Nature" in 1976...»*

He had plans to write a book on **"A Descriptive Theory of Deterministic Chaos"**.

He left on the blackboard in his office at the Institute of Mathematics the inscription: "Do not wipe!". Also, on the same board was written by him the contents of his planned book:

A Descriptive Theory of Deterministic Chaos

1. Introduction.

1.1.Descriptive theory of set.

1.2.Dynamical systems and chaos.

1.3. Attractors of trajectories and their basins.

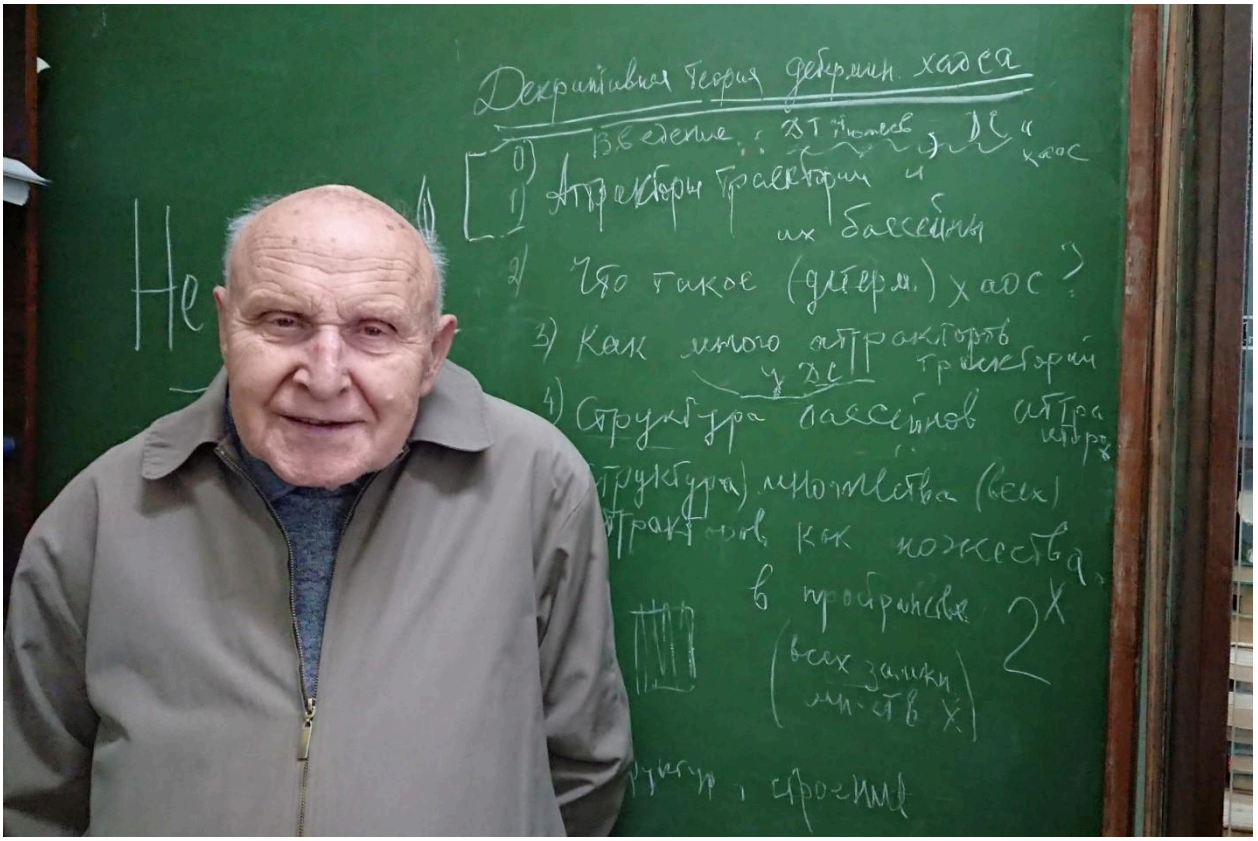
2. What is a Deterministic chaos.

3. How many attractors of trajectory does a dynamical system have.

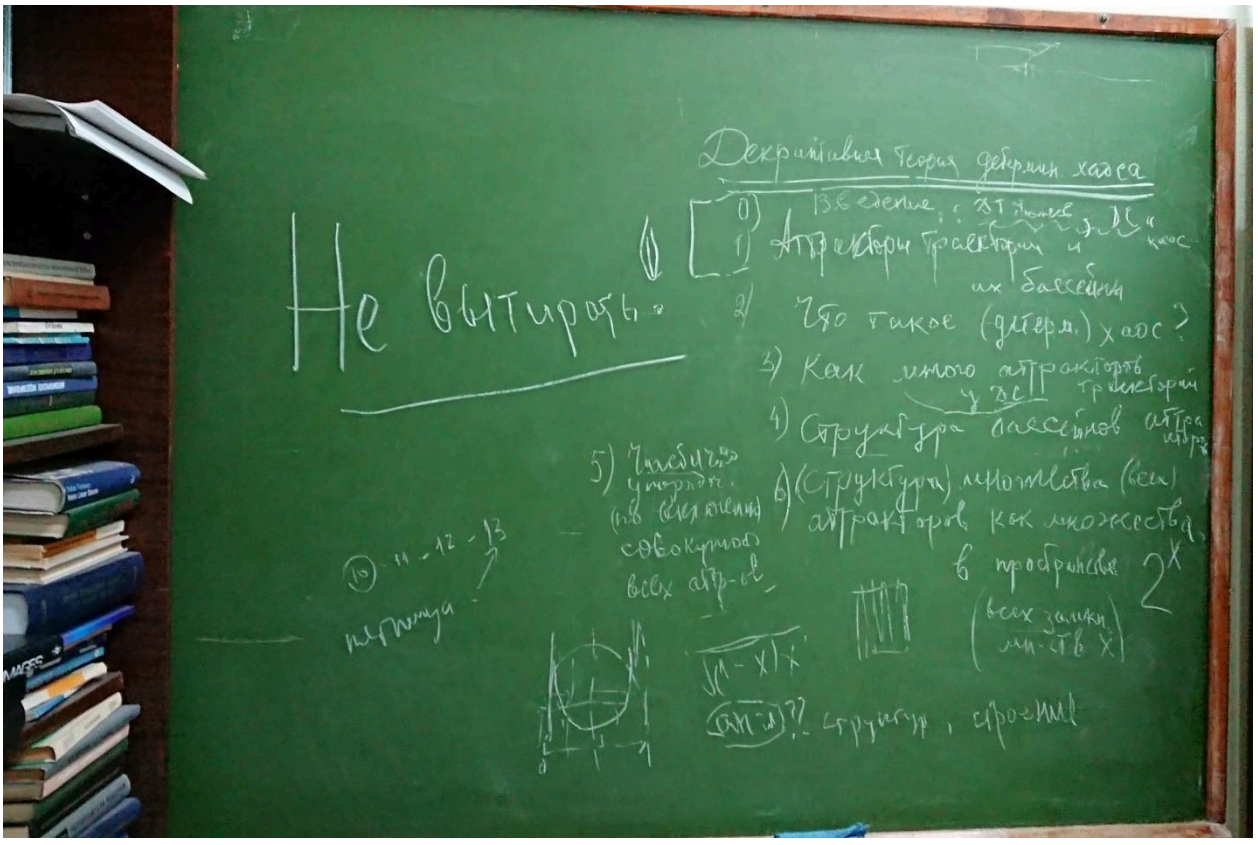
4. The structure of basins of attractors.

5. Partially ordered (by inclusion) set of all attractors.

6. Structure of the set of (all) attractors as a set in 2x space.



*O.M. Sharkovsky in front of a blackboard in his office at the Institute of Mathematics, on the blackboard written contents of the planned book **"A Descriptive Theory of Deterministic Chaos"***



*The contents of the planned book **"A Descriptive Theory of Deterministic Chaos"** and the call "Do not wipe!" on the board.*

For the last 3 years 2020 -2022 Oleksandr Sharkovsky has been working on three articles on this topic.

The article ***"Descriptive Theory of Deterministic Chaos"***, *Ukr Math J, Vol. 74, July 2023, pp. 1950–1960, doi.org/10.1007/s11253-023-02180-z (transl. from Ukrains'kyi Matematychnyi Zhurnal, Vol. 74, no. 12, Jan. 2023, pp. 1709 -18)*, written by him, was submitted to the editorial board of the Ukrainian Mathematical Journal in Spring 2021, but for reasons beyond his control, it was published only after he passed away in December 2022.

The second article on this topic ***"Locally Maximal Attractors of Expanding Dynamical Systems***", *Ukrains'kyi Matematychnyi Zhurnal, Vol. 76, no. 1, Feb. 2024, pp. 17 -30 (in Ukrainian)*, was prepared by him with co-authors Vasily Bondarchuk and Andrey Sivak in 2021, but for the same reasons was not published during his lifetime. This paper was published by Ukrainian Mathematical Journal in January 2024.

In the article ***"Descriptive Theory of Deterministic Chaos"*** he mentioned that: «In the S. Ruette monograph "Chaos on the Interval", published in early 2017, for continuous interval maps proposed a survey of relations between different types of chaos and related notions. For the author of this paper, S. Ruette's monograph became the last and decisive impulse for understanding the results he obtained back in the middle of the 1960s and analyzing the results in the papers ***"On attracting and attracted sets"*** (1965), ***"A classification of fixed points"*** (1965), and "***Behavior of a mapping in the neighborhood of an attracting set"*** (1966) from the point of view of "chaos".»

The December 2023 issue of the Ukrainian Mathematical Journal (*Vol 75 No 12 2023*) was an anniversary issue dedicated to O.M. Sharkovsky, and the Ukrainian Mathematical Journal (*Vol 76 No 1 2024*) issue in January 2024 was dedicated to the 60th anniversary of Sharkovsky's theorem.

His third planned article was ***"Basins of attractors of trajectories of Expanding Dynamical Systems"***. The third article (continuation of the second article) written with co-authors is being prepared for publication.

Oleksandr Sharkovsky left behind a substantial archive of material on the topic of chaos.

Olena Sharkovska

The personal Oleksandr Mykolayovych Sharkovsky web-page:
https://www.imath.kiev.ua/~asharkov/